Transport & acceleration of space charge dominated beam with Cyclotron

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Plan of the talk

- Introduction
- Brief discussion on cyclotrons
- Space charge effects
- Transverse space charge effects
- Longitudinal space charge effects
- Summary

Reference: M. Reiser, "Theory and Design of Charged Particle Beams", John Wiley and Sons, New York (1994).

Introduction

High intensity accelerators are needed for applications like

- Spallation neutron sources
- Hybrid reactor system (energy amplifier)

Aim of a high intensity accelerator :

- A beam with desired current and energy be delivered to the target.

The central issue during the transport and acceleration process:

- There should be no appreciable loss of particles.
- There should be no excessive emittance growth.

Activation of the components must be within tolerable limit.

Space charge effect is a major problem in accelerators and transport lines.

Space charge effects in cyclotron

Transverse space charge effect:

- Increases the beam size and reduces vertical focusing frequency

  $\Rightarrow$ axial beam loss (serious at low energies)

- Strongest on the first few turns (energy is low and focusing forces are small.)

Longitudinal space charge effect:

- Increases the energy spread & expands the radial region of each bunch

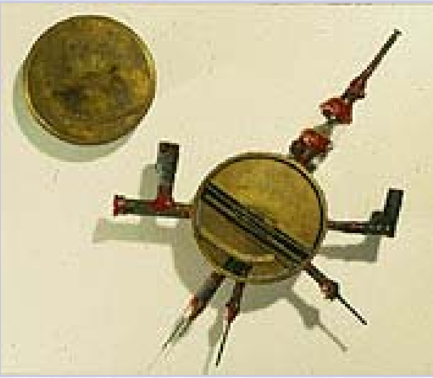
  $\Rightarrow$ reduction in the turn separation

  $\Rightarrow$ extraction loss

- It is of concern throughout because (no longitudinal focusing in the cyclotrons.)

What is a cyclotron ?

Cyclotron (Lawrence, 1929)



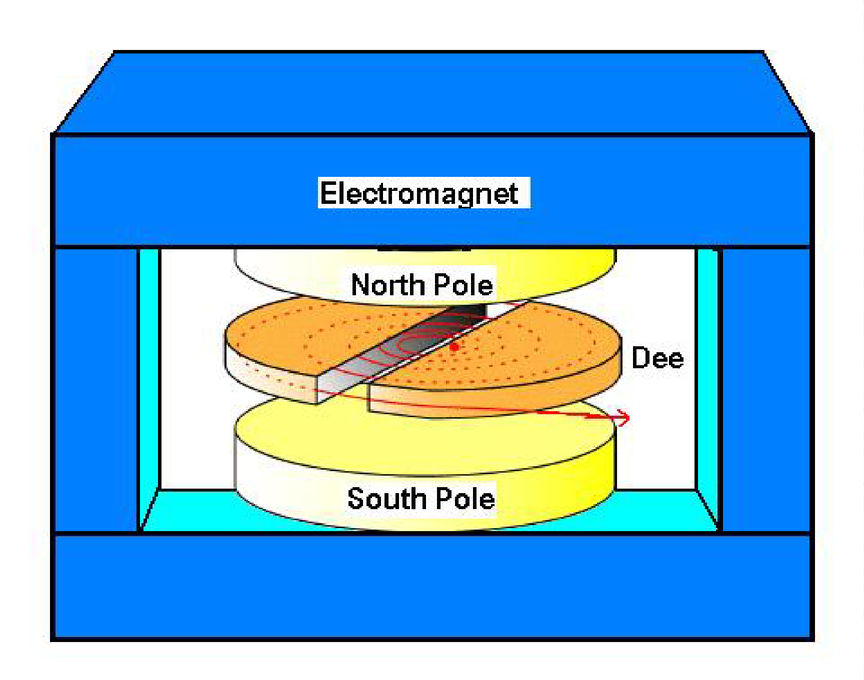
- Based on the principle of
  - bending of charge particles in a magnetic field.
  - radio frequency acceleration
- Same electrode is utilized again and again to accelerate particles.

Kinetic energy of the particles

$$T = \frac{1}{2} m v_0^2 = \frac{q^2 B^2 R^2}{2m} = K \frac{Q^2}{A}$$

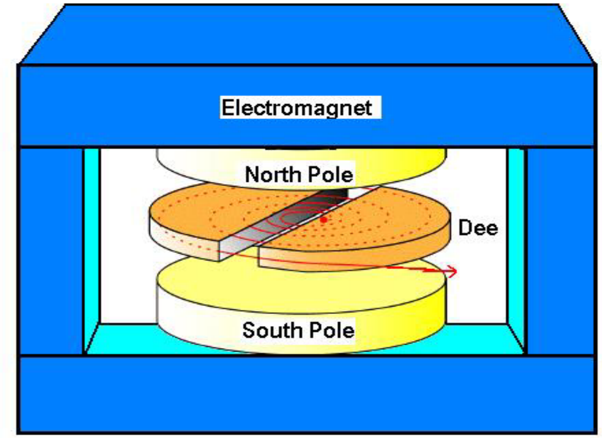
$$T = n \times 2 q V_D$$

$$K = \frac{e^2 B^2 R^2}{2 m_p}$$



Basic equations

$$\frac{mv^2}{r} = qvB \qquad \omega_p = \frac{qB}{m} \qquad p = qBr$$



Orbital frequency is independent of the particle energy.

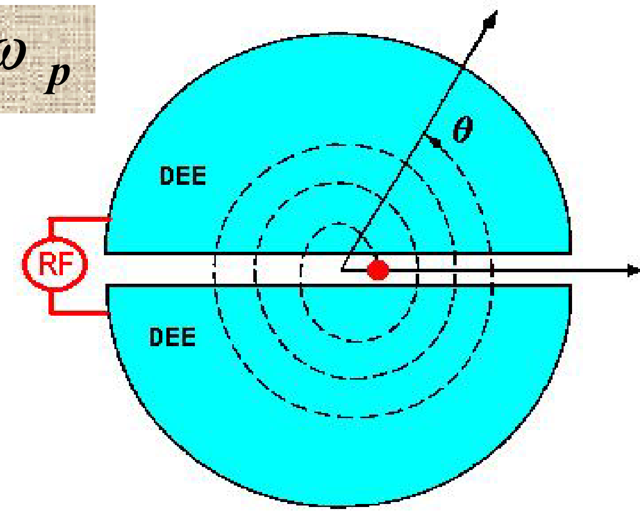
Orbit radius is proportional to the particle momentum.

Repeated acceleration needs resonance $\omega_{rf} = h\omega_p$

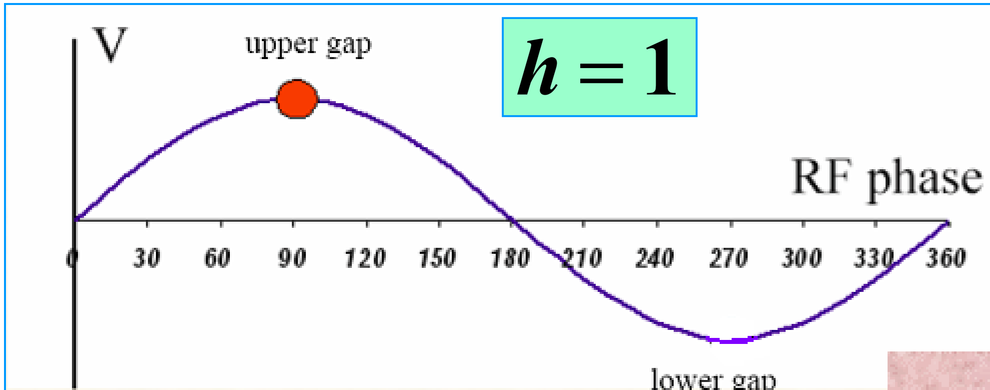
A discrepancy in resonance $\Rightarrow$ phase shift

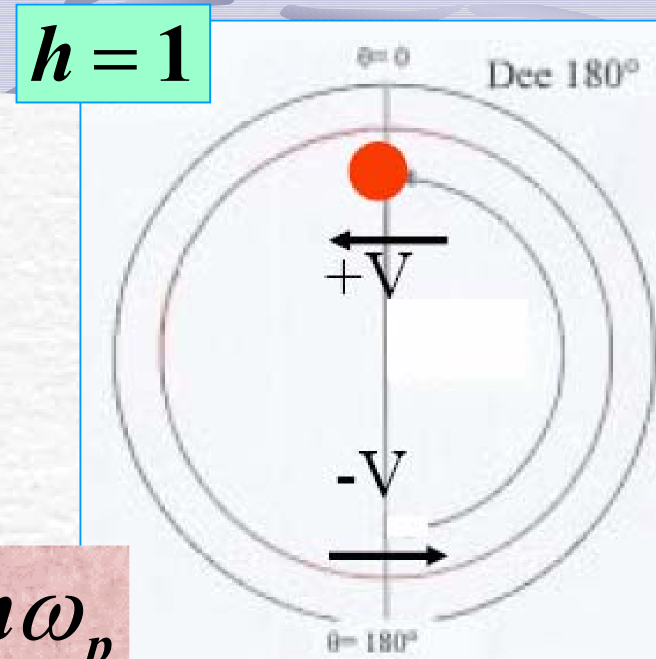
$$d(\sin\phi) = 2\pi hn \frac{\omega_{rf} - \omega}{\omega}$$

Limit of acceleration occurs at $\phi = \pm 90^0$

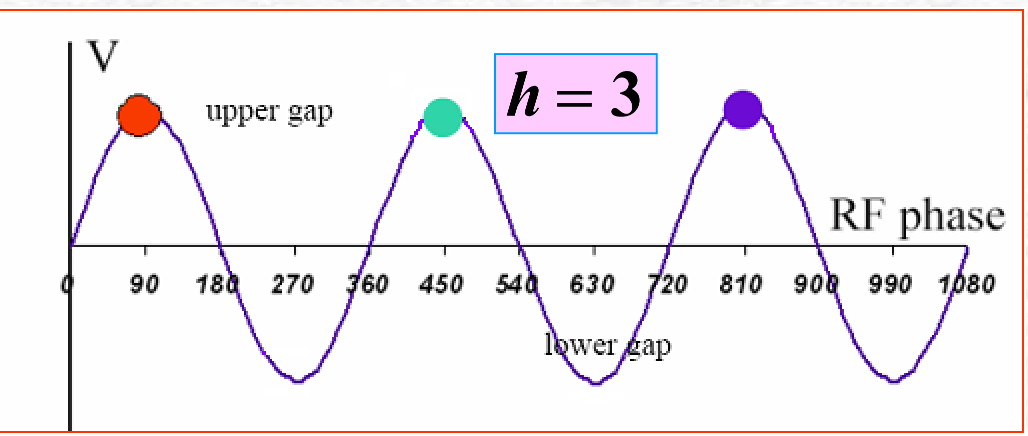


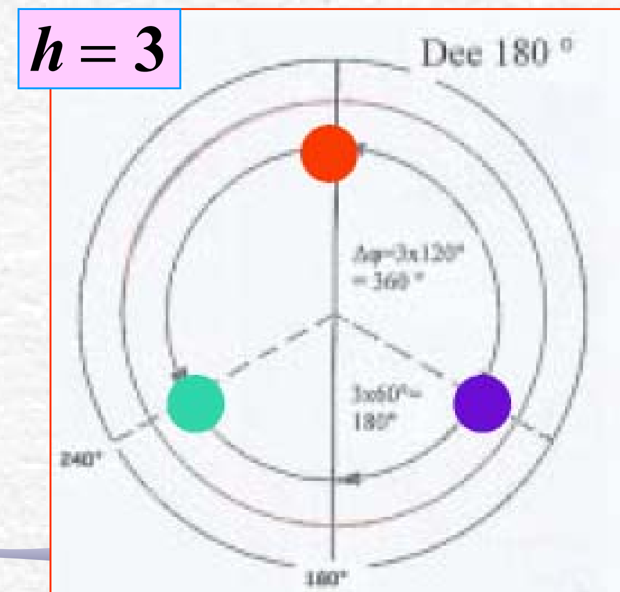
Harmonic Number h





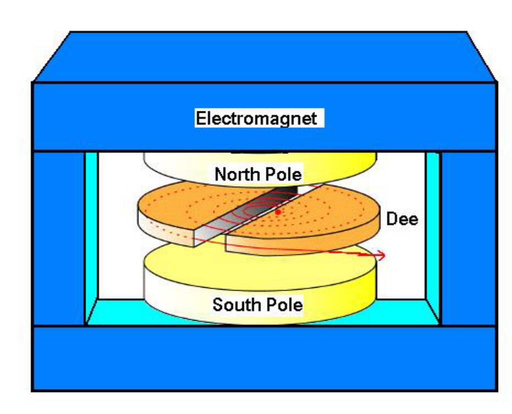
$$\omega_{rf} = h\omega_p$$





Orbit Stability

- In a uniform field particles orbits do not have axial stability.
- A particle with a small axial velocity will soon strike the dee chamber and be lost.



- In a nominal beam current (μA), there are trillions of particles that repel each other.

- It is necessary to provide axial focusing to the particles, which have an upward or downward velocity component.

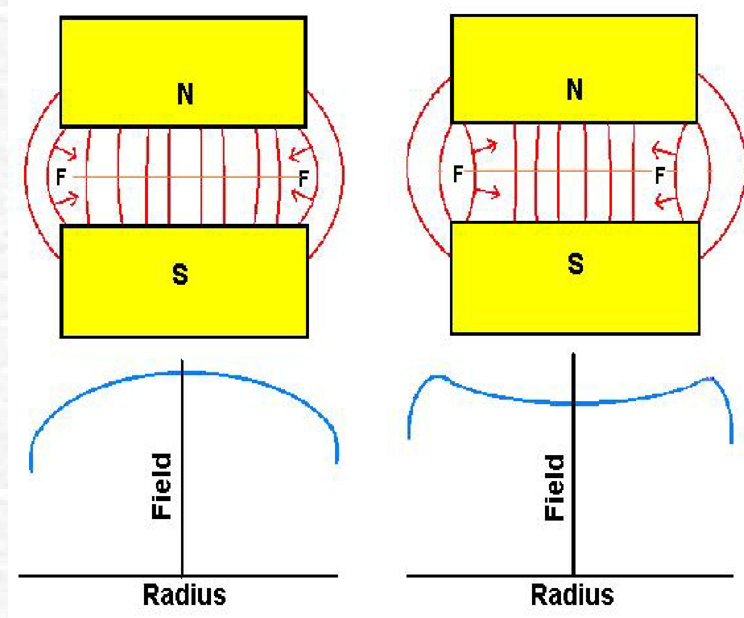
- It is the problem of vertical focusing which led to the developments of many kinds of cyclotrons
  - Azimuthally varying field cyclotron
  - Frequency modulated cyclotron
  - Microtron

Classical Cyclotrons

$s = vt = r\theta \qquad (r, s, z)$

$$x'' = \frac{\partial^2 x}{\partial s^2} = \frac{1}{R^2}\frac{\partial^2 x}{\partial \theta^2} = \frac{1}{v^2}\frac{\partial^2 x}{\partial t^2}$$

- Small machines were built.
- Average magnetic field decreased with radius



Off orbit particles execute SHM around EO

$$z'' + \frac{\upsilon_z^2}{R^2} z = 0 \qquad x'' + \frac{\upsilon_r^2}{R^2} x = 0$$

- Betatron tunes determine the orbit stability

$$\upsilon_z^2 = n \qquad \upsilon_r^2 = 1 - n \qquad n = -\frac{r}{B}\frac{dB}{dr}$$

Stability in both planes $\Rightarrow$ Real tunes $\quad 0 \le n \le 1 \quad B\downarrow r\uparrow$

- Very weak focusing
- Against resonance condition
- Unsuitable for relativistic particles

$$\omega_{rf} = \omega_p = \frac{qB}{m}$$

AVF Cyclotron

$s = vt = r\theta$ (r, s, z)

Average field is increased with radius to counter relativistic effect

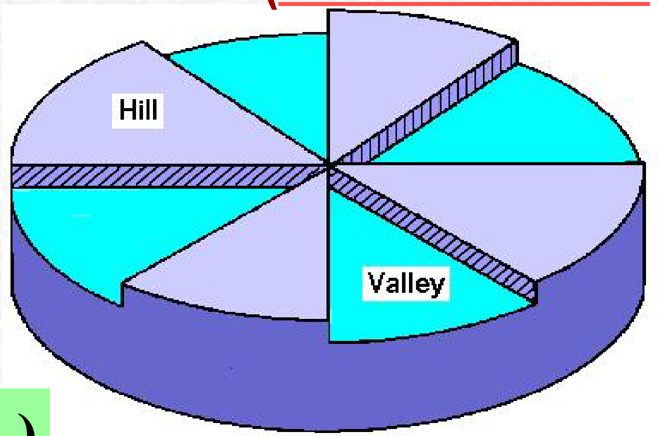
$$\omega_{rf} = h\frac{qB}{\gamma m}$$

Off orbit particles execute SHM

$$B(r) = \gamma(r) B_0$$

(L.H. Thomas 1938

$$z'' + \frac{\upsilon_z^2}{R^2} z = 0 \qquad x'' + \frac{\upsilon_r^2}{R^2} x = 0$$



- Straight Sector cyclotron

$$\upsilon_z^2 = -(\gamma^2 - 1) + F^2$$

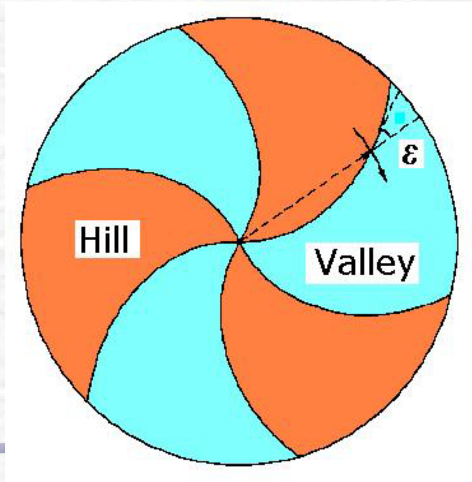
$$\upsilon_r^2 = \gamma^2 + ..$$

$$F^2 = \frac{(B_H - \overline{B})(\overline{B} - B_V)}{\overline{B}^2}$$

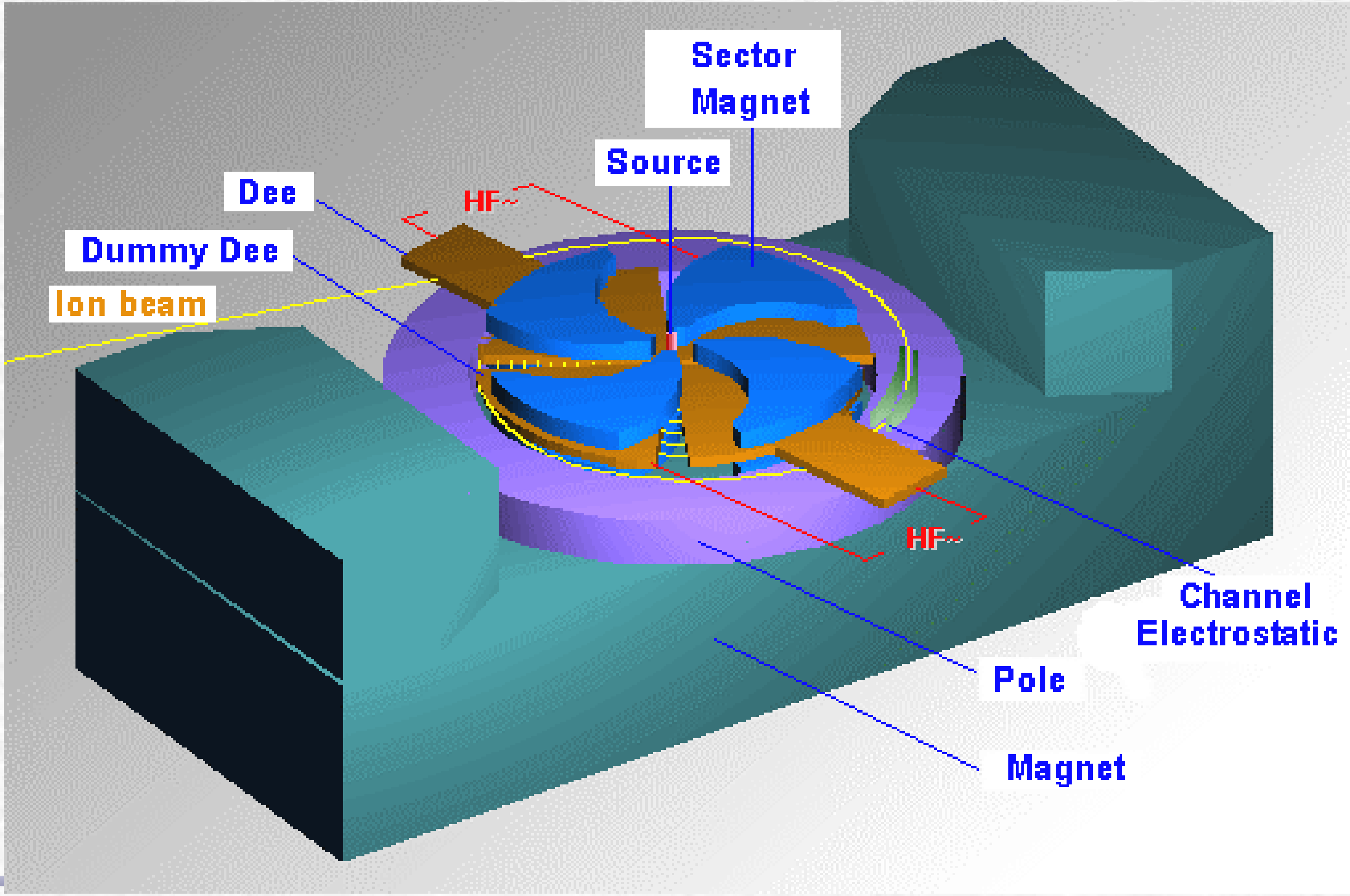
- Spiral Sector cyclotron

$$\upsilon_z^2 = -(\gamma^2 - 1) + F^2(1 + 2\tan^2 \varepsilon)$$

$$\upsilon_r^2 = \gamma^2 + ..$$



A Typical AVF Cyclotron



Extraction From Cyclotron

- Electrostatic Channel

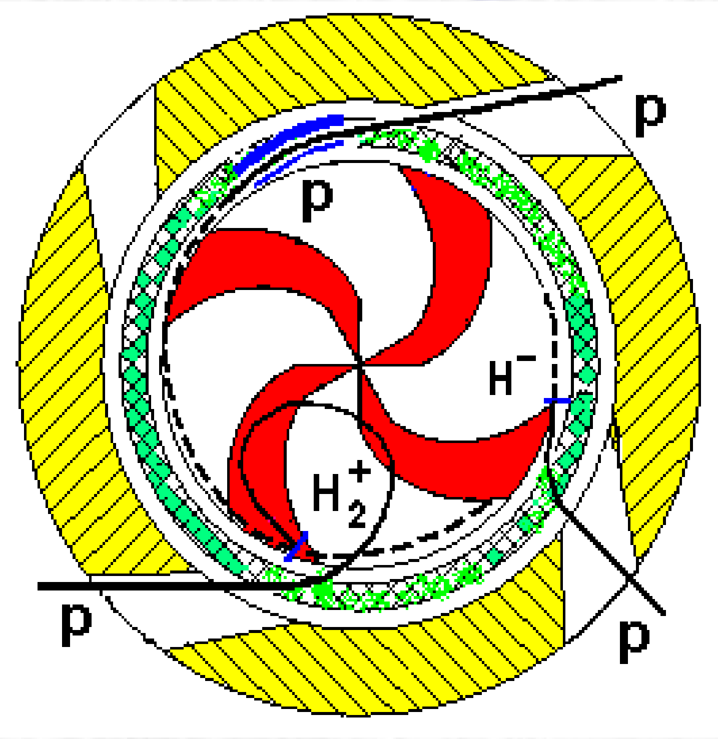
$$qvB - F_{out} = \frac{mv^2}{R}$$

A clear turn separation is needed at extraction.

$$\frac{dR}{dn} = R\frac{\Delta T}{T}\frac{\gamma}{\gamma+1}\frac{1}{\nu_r^2}$$

Make cyclotron with large radius R.

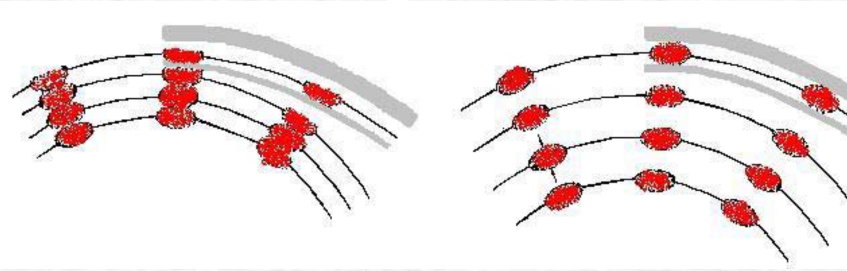
Provide high energy gain per turn.



- Extraction by stripping for H^- H_2^+

A thin foil is inserted at a suitable radius.

Stripping changes the radius of curvature.

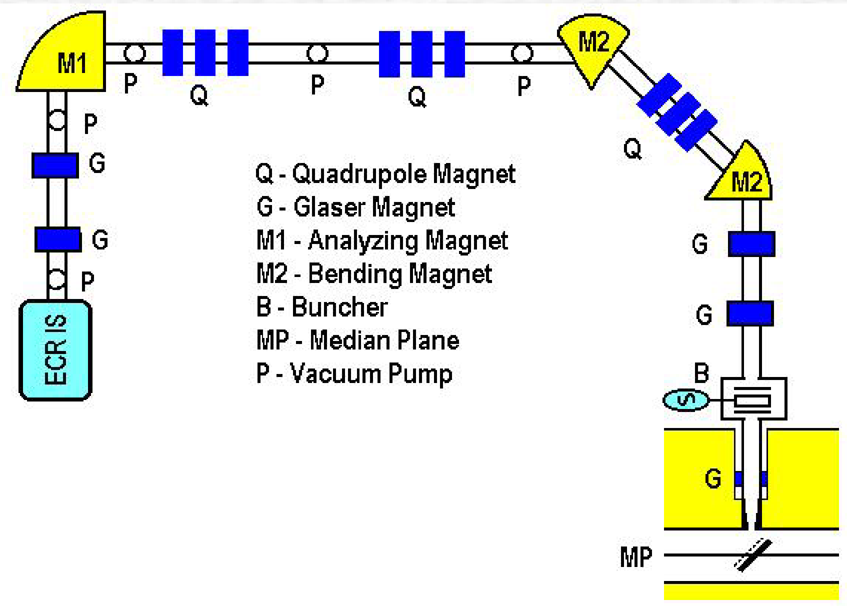


VECC at Kolkata

K=130 MeV
N = 3 (spiral)
Dee = 1

protons: 6-60 MeV
deuterons: 12-65 MeV
heavy ions: 130 Q^2/A MeV.

VEC provides:
- light ions internal PIG ion source
- heavy ions with ECR ion source



Research in: Nuclear Physics, Condensed Matter Physics, Material Sciences

Superconducting Cyclotron

Magnetizing force is supplied by sc-coils, consuming little power.

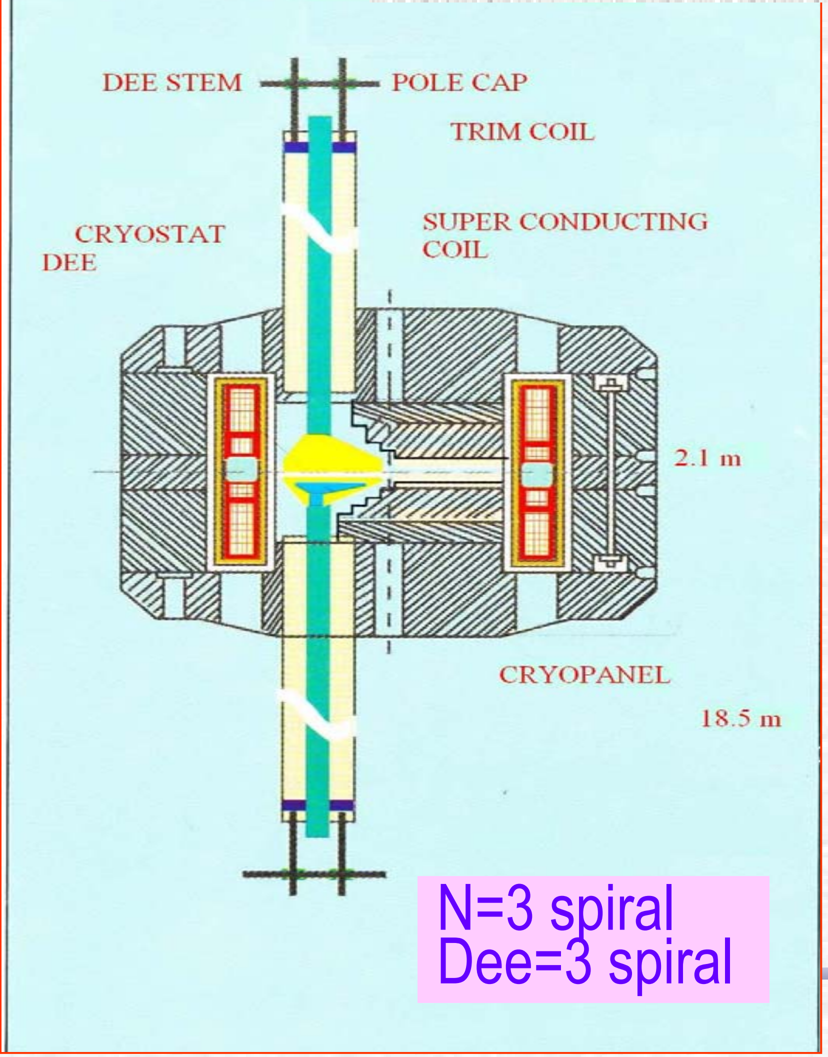
SC coils ~ NbTi
High B ~ 5-6 Tesla

$$T = \frac{q^2 B^2 R^2}{2m}$$

$$B = \mu_0 (H + M)$$
$$H \; \alpha \; NI$$

K-500 at MSU
K-500 at VECC

Ext. radius =67cm
Iron wt. = 100 tons

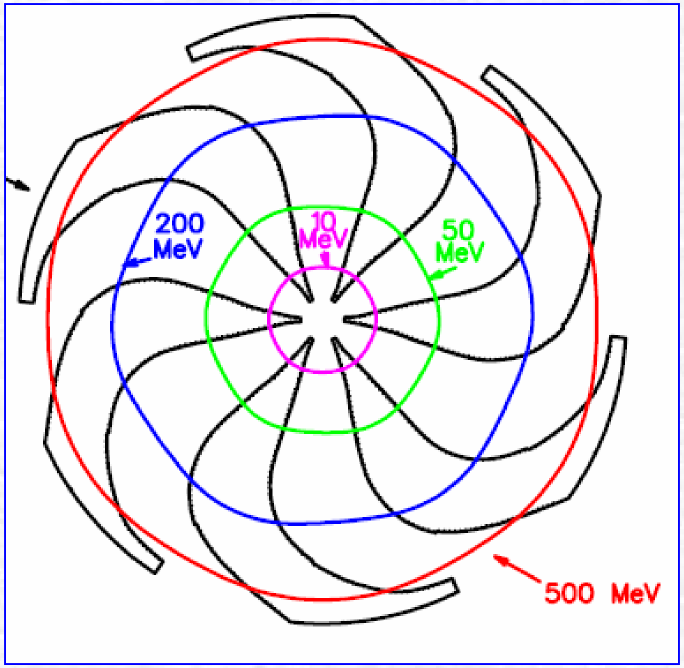


N=3 spiral
Dee=3 spiral

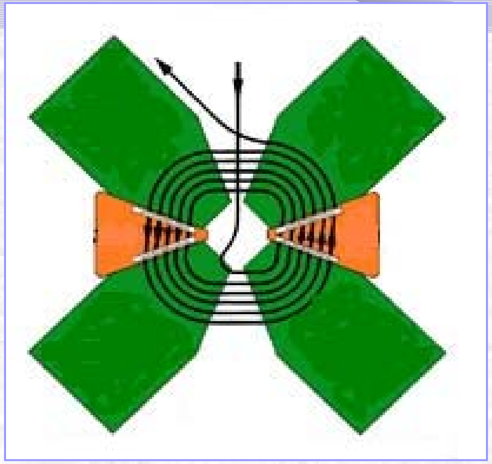
Largest AVF Cyclotron

- TRIUMF : protons 520 MeV for pion production
- H^- ions are injected from external ion source.
- Extraction: by stripping (Carbon foil)

N=6 spiral
dee = 2, 180 deg
Diameter =12m
Iron wt. = 4000 tons



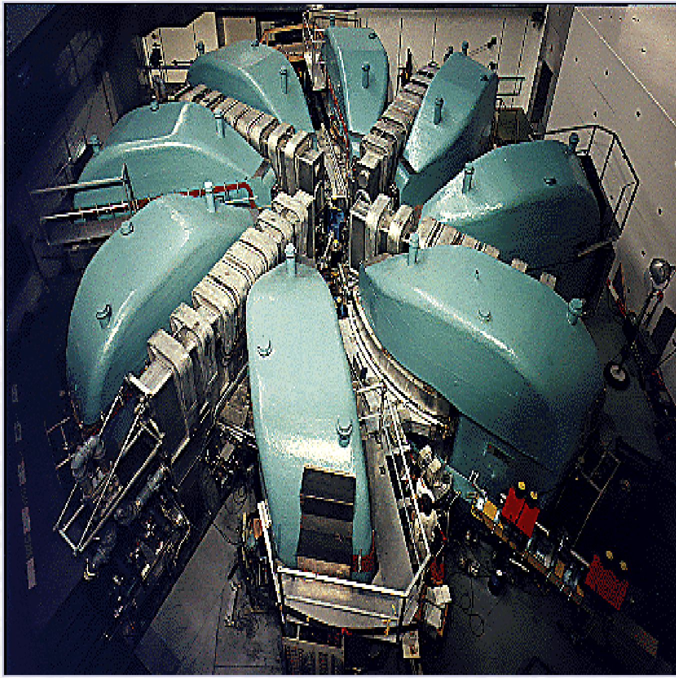
Separated Sector Cyclotron



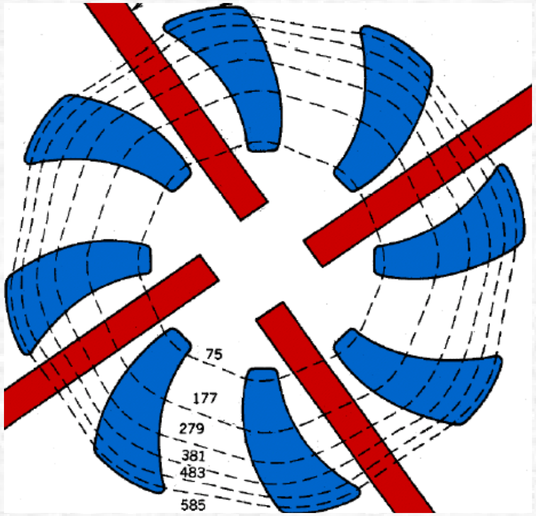
Magnet sectors are separated by empty valleys.
Used for high current.

$$\Delta \nu_z^2 = F^2 = \frac{(B_H - \overline{B})(\overline{B} - B_V)}{\overline{B}^2}$$

Focusing force will be max when *Bv* = 0.
RF structures are put between the sectors



PSI Machine:
Energy ~ 590 MeV p
Current ~ 2mA



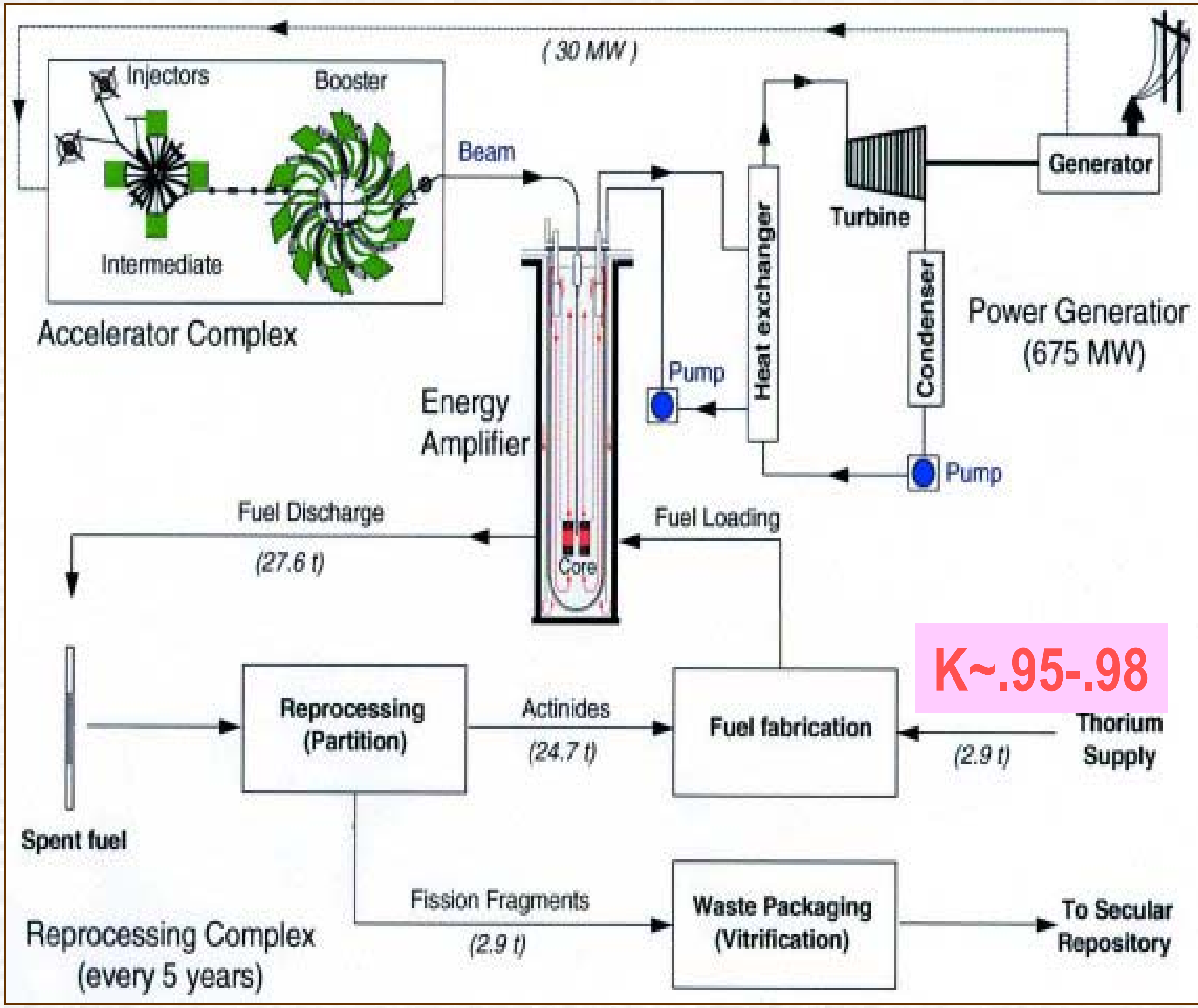
Used for the production of
pions
spallation neutrons.

ADSS for Energy Production

A sub-critical device with ^{232}Th - ^{233}U fissile core

No Plutonium
No actinide waste

^{232}Th is abundant.
Could last >1000years.



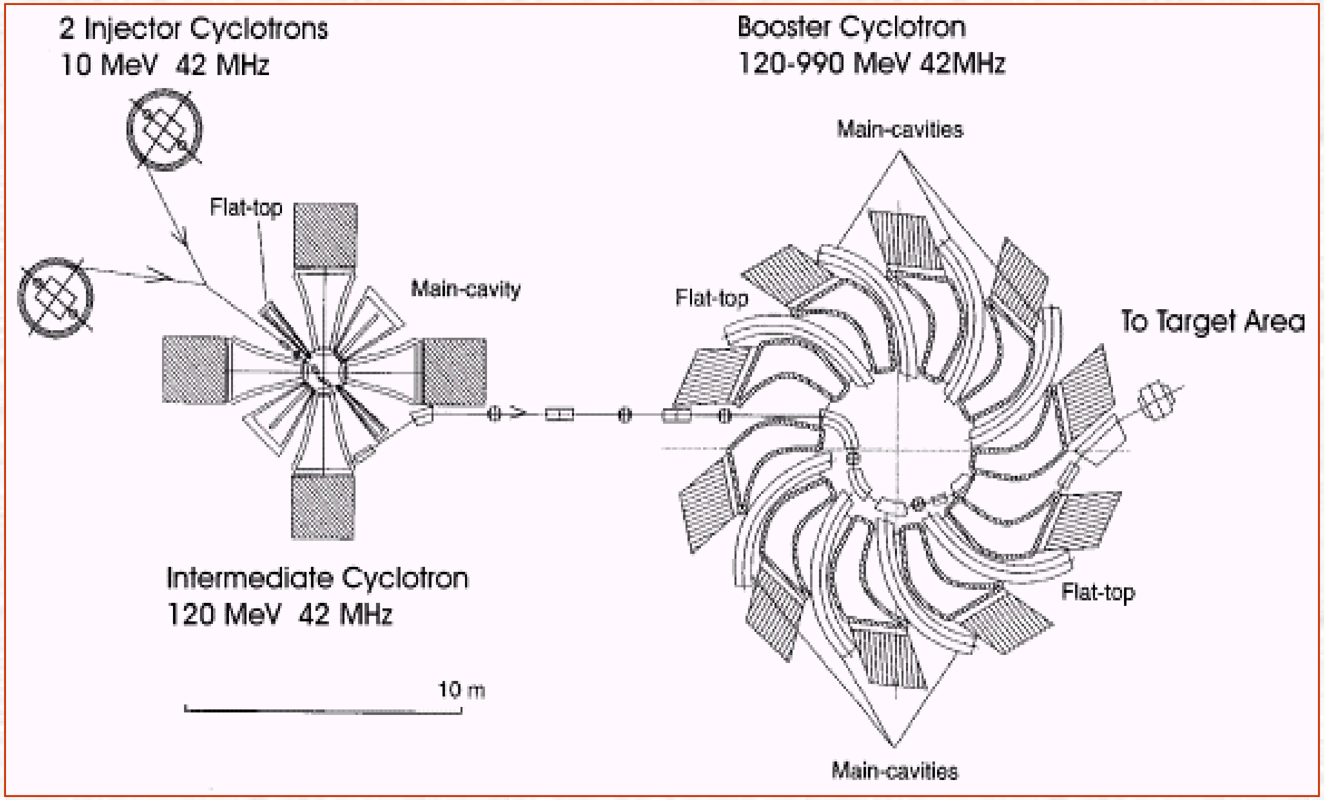
K~.95-.98

Cyclotron option for ADSS

Requirement:

$E_P \sim 1$ GeV

$I \sim 10$mA

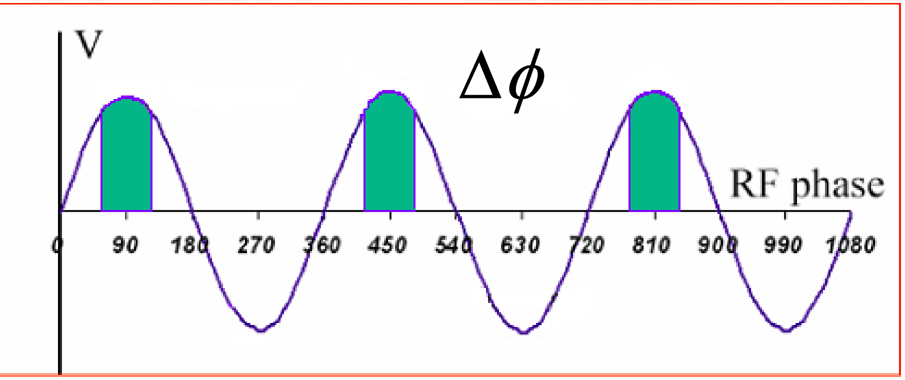


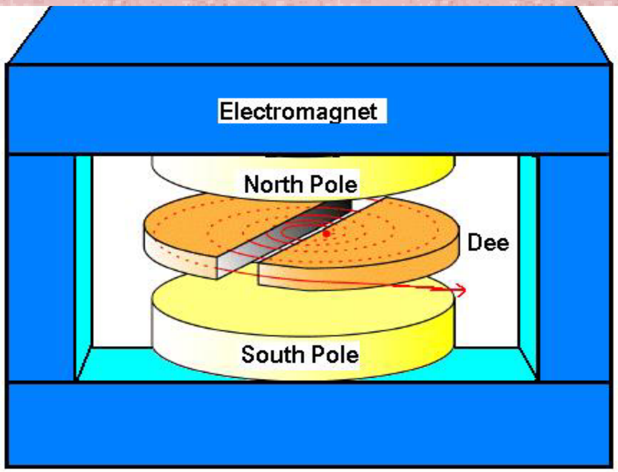
Only Separated Sector Cyclotron can handle such high beam current

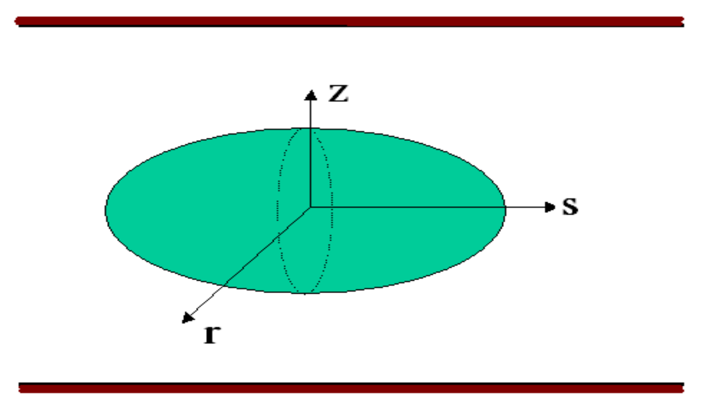
Beam from a cyclotron

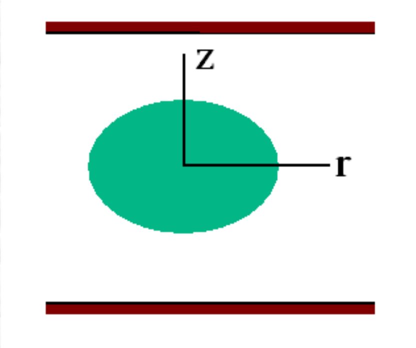
(r, s, z) $s = vt = r\theta$

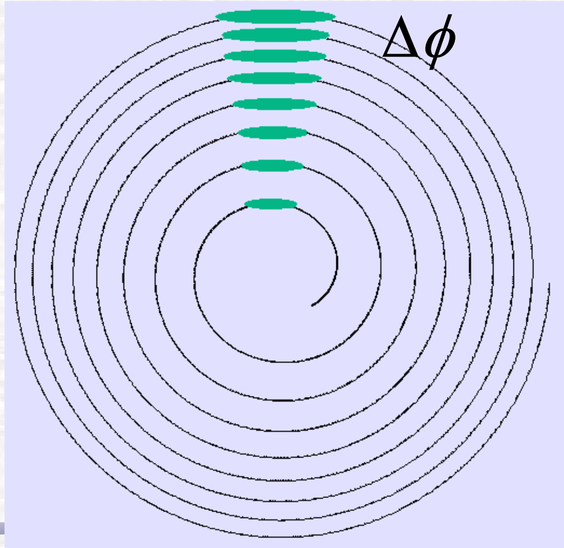
$$x'' = \frac{\partial^2 x}{\partial s^2} = \frac{1}{R^2}\frac{\partial^2 x}{\partial \theta^2} = \frac{1}{v^2}\frac{\partial^2 x}{\partial t^2}$$











$$Ip = qNf = \frac{qN\omega}{2\pi}$$

$$V = \frac{4}{3}\pi abc$$

$$\rho = \frac{Q}{V} = \frac{3Nq}{4\pi abc}$$

$$I * 2\pi = Ip * \Delta\phi$$

What is space charge effect

Moving charges produce

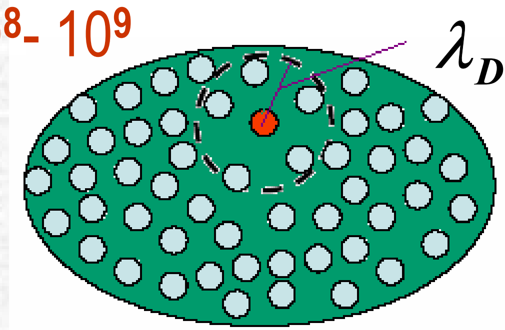
mutually repulsive electric field.

attractive magnetic field (small for v<<c)

The total effect on any particle is the sum of the fields due to all particles.

Summing the fields directly from all particles ???

n~10^8- 10^9



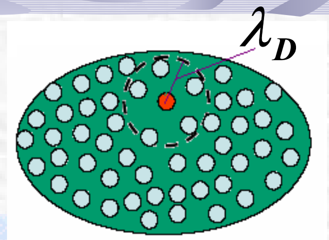
$$\lambda_D = \sqrt{\frac{\gamma \varepsilon_0 kT}{nq^2}}$$

The net effect of the Coulomb interactions can be classified into

- Collisional Regime (Single Particle Effects) : dominated by binary collisions caused by close particle encounters., Bunch size < λ_D
- Space Charge Regime (Collective Effects) : dominated by the self field produced by the particle distribution, that can be represented by a smooth field as a function of space and time: Bunch size >> λ_D

Space Charge Effect

For an ellipsoidal bunch with projection ***a***



100keV p a $= 5mm$

$\varepsilon_n = .2\pi \; mmmrad$

$$\lambda_D = \sqrt{\frac{\gamma\varepsilon_0 kT}{nq^2}} = \frac{c\varepsilon_n}{2}\sqrt{\frac{\pi\varepsilon_0 m\beta\gamma}{qI}\cdot\frac{2\pi}{\Delta\phi}}$$

I(mA)	λ_D(mm)	N/cc
0.1	1.06	1.8×10^{7}
1	0.337	1.8×10^{8}
10	0.107	1.8×10^{9}
30	0.06	5.4×10^{9}
100	0.035	1.8×10^{10}

The effective interaction range of the test charge is limited to the **Debye length**

In the absence of collisions,

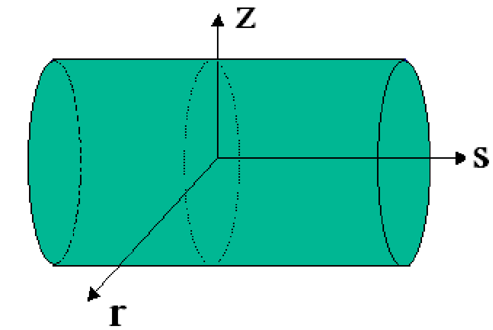
1. smooth functions for charge and field distributions can be used.
2. forces can be treated like an applied force.
3. phase space volume (6D) remains constant during the acceleration.

If all forces are linear and no coupling

normalized emittance in each 2-D remains a constant of the motion

Transverse Space Charge Effects

Uniform cylindrical beam circular cross section



$$\rho = \frac{I}{\pi a^2 \beta c}$$

$$J = \frac{I}{\pi a^2}$$

Due to symmetry we have only

radial electric field

azimuthal magnetic field

Maxwell's equations

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{r}\frac{\partial}{\partial r}(rE_r) = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{r}\frac{\partial}{\partial r}(rB_\theta) = \mu_0 \vec{J}$$

$$E_r = \frac{\rho r}{2\varepsilon_0} = \frac{Ir}{2\pi\varepsilon_0 a^2 \beta c}$$

$$B_\theta = \frac{\mu_0 J r}{2} = \frac{Ir}{2\pi\varepsilon_0 c^2 a^2}$$

$$E_r = \frac{\beta}{c} B_\theta$$

$$F_r = q(E_r - \beta c B_\theta) = q(1-\beta^2)E_r = \frac{qE_r}{\gamma^2}$$

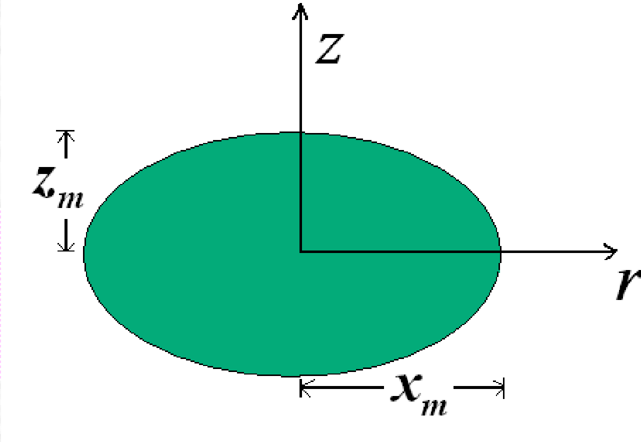
Force is linear, defocusing

Strong for v<<c

a non-relativistic effect

Transverse Space Charge Effects

Uniform cylindrical beam elliptical cross section



The electrostatic potential in the beam frame

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} = -\frac{\rho}{\varepsilon_0}$$

$$\rho = \frac{I}{\pi x_m z_m \beta c}$$

Solution of potential inside the beam

$$\phi(x,z) = -\frac{\rho}{4\varepsilon_0}\left[x^2 + z^2 - \frac{x_m - z_m}{x_m + z_m}(x^2 - z^2)\right] + cont$$

$$E_x = -\frac{\partial\phi}{\partial x} = \frac{I}{\pi\varepsilon_0\beta c}\cdot\frac{x}{x_m(x_m + z_m)}$$

$$E_z = -\frac{\partial\phi}{\partial x} = \frac{I}{\pi\varepsilon_0\beta c}\cdot\frac{z}{x_z(x_m + z_m)}$$

Force in the lab frame

$$F_{x,z} = \frac{1}{\gamma^2}qE_{x,z}$$

(Lorentz transformation)

T SC Effects: Tune shift

Recalling the Hill's equation

$$x''(s)+k(s)x=0$$

$$x''=\frac{d^2x}{ds^2}=\frac{1}{v^2}x\;[\text{illegible}]\;\frac{1}{\gamma m\beta^2c^2}F_x$$

Including the space charge force

$$x''(s)+k(s)x=\frac{1}{\gamma m\beta^2c^2}F_x=\frac{qI}{\pi m\varepsilon_0\beta^3c^3\gamma^3}\cdot\frac{x}{x_m(x_m+z_m)}=\frac{2K\cdot x}{x_m(x_m+z_m)}$$

K=generalized perveance

$$K=\frac{q}{4\pi m\varepsilon_0c^3}\cdot\frac{2I}{\beta^3\gamma^3}=\frac{1}{I_0}\cdot\frac{2I}{\beta^3\gamma^3}$$

$$I_0=\frac{A}{Q}\cdot\pi\cdot10^7\,A$$

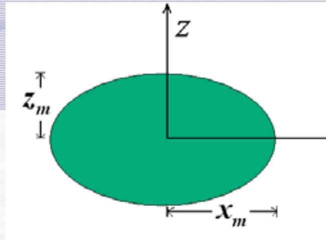
Equations of motions in both planes

$$x''(s)+\left(k_x(s)-\frac{2K}{x_m(x_m+z_m)}\right)x=0$$

$$z''(s)+\left(k_z(s)-\frac{2K}{z_m(x_m+z_m)}\right)z=0$$

Equations are coupled via space charge term

TSC Effects : Tune shift



For a symmetric beam $x_m = z_m = a$

In a circular accelerator having smooth focusing $\bar{k}_{x,z}(s) = \frac{\upsilon_{r,z}^2}{R^2} = \frac{\upsilon_0^2}{R^2}$

Equations of motions

$$x''(s) + \left(\frac{\upsilon_0^2}{R^2} - \frac{K}{a^2}\right)x = x''(s) + \frac{\upsilon^2}{R^2}x = 0$$

$$\upsilon^2 = \upsilon_0^2 - \frac{KR^2}{a^2}$$

Direct space charge force leads to defocusing $\Rightarrow$ lowering of betatron tunes

Change in tune value

$$\Delta\upsilon = \upsilon_0 - \upsilon = \frac{KR^2}{2\upsilon_0 a^2} = \frac{KR}{2\varepsilon}$$

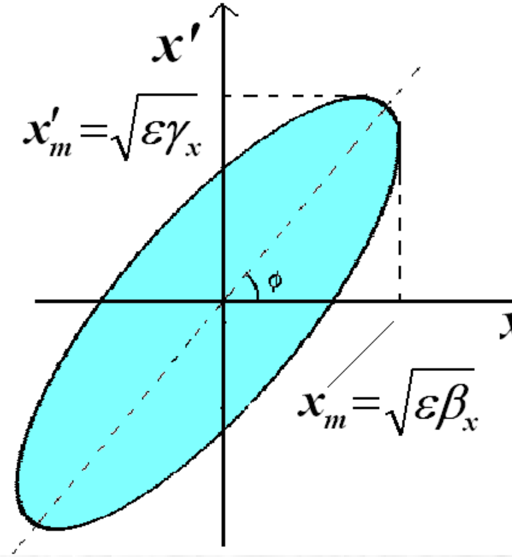
$$\varepsilon_n = \beta\gamma\varepsilon = \beta\gamma\frac{\upsilon_0 a}{R}$$

For a bunched beam

$$\Delta\upsilon = \frac{I}{I_0}\cdot\frac{R}{\varepsilon_n}\cdot\frac{1}{\beta^2\gamma^2}\cdot\frac{2\pi}{\Delta\phi}$$

$$K = \frac{2I}{I_0\beta^3\gamma^3}$$

T SC Effects: Envelope Equations



Recalling the Hill's equation $x''(s) + k(s)x = 0$

With solutions $x(s) = \sqrt{\varepsilon_x}\,\omega(s)\cos\left(\psi(s) + \varphi_0\right)$

Amplitude function ω satisfies the relation $\omega'' + k(s)\omega - \dfrac{1}{\omega^3} = 0$

Beam envelope is characterized by $x_m = \sqrt{\varepsilon\omega} = \sqrt{\varepsilon\beta_x}$

Beam envelope equation is $x''_m + k(s)x_m - \dfrac{\varepsilon_x^2}{x_m^3} = 0$

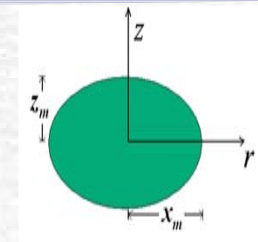
$K = \dfrac{2I}{I_0\beta^3\gamma}$

For the beam with space charge coupled envelope equations are

$$x''_m + k_x(s)x_m - \frac{2K}{x_m + z_m} - \frac{\varepsilon_x^2}{x_m^3} = 0 \qquad z''_m + k_z(s)z_m - \frac{2K}{x_m + z_m} - \frac{\varepsilon_z^2}{z_m^3} = 0$$

T SC Effects: Envelope Equations

For a symmetric beam in smooth focusing condition $x_m = z_m = a$

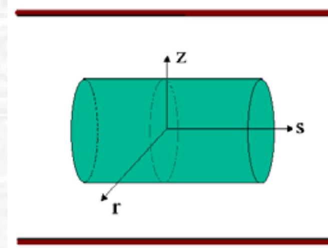


$$a'' + \frac{\upsilon_0^2}{R^2} a - \frac{K}{a} - \frac{\varepsilon^2}{a^3} = 0$$

$$\overline{k}(s) = \frac{\upsilon_r^2}{R^2} = \frac{\upsilon_0^2}{R^2}$$

For a special case when $a' = 0$

$$\varepsilon_x = \varepsilon_z = \varepsilon$$



$$\frac{\upsilon_0^2}{R^2} a - \frac{K}{a} - \frac{\varepsilon^2}{a^3} = 0$$

1. Emittance dominated beam $Ka^2 << \varepsilon$

Matched beam radius $a_0^2 = \frac{\varepsilon R}{\upsilon_0}$

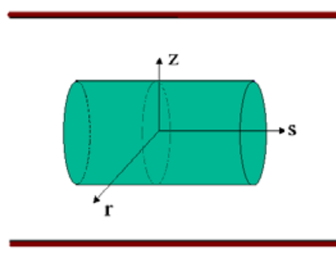
2. Space charge dominated beam $Ka^2 >> \varepsilon$

Beam radius $a^2 = \frac{KR^2}{\upsilon_0^2}$

T SC Effects: limiting current

Beam radius is determined by the available aperture in the machine.

$$\frac{\upsilon_0^2}{R^2}a - \frac{K}{a} - \frac{\varepsilon^2}{a^3} = 0$$



Acceptance $$\varepsilon_m = a_m^2 \frac{\upsilon_0}{R}$$

$$K = \frac{2I}{I_0\beta^3\gamma^3}$$

The maximum value of K_m and hence I_m can be obtained as

$$K_m = \frac{\upsilon_0^2 a_m^2}{R^2}\left(1 - \frac{\varepsilon^2}{\varepsilon_m^2}\right)$$

$$I_m = \frac{I_0}{2}\frac{\beta^3\gamma^3\upsilon_0^2 a_m^2}{R^2}\left(1 - \frac{\varepsilon^2}{\varepsilon_m^2}\right)$$

$$a_0^2 = \frac{\varepsilon R}{\upsilon_0}$$

For a bunched beam and small emittance, Limiting current is

$$\varepsilon_n = \beta\gamma\varepsilon$$

$$I_m = \frac{I_0}{2}\beta^2\gamma^2\varepsilon_n\frac{\upsilon_0}{R}\cdot\frac{\Delta\phi}{2\pi}$$

TSC effects dominate at low velocities and strongly bunched beam

T SC Effects: tune shift and beam radius

We can solve for increase in beam radius with beam current using

$$\frac{\upsilon_0^2}{R^2} a - \frac{K}{a} - \frac{\varepsilon^2}{a^3} = 0$$

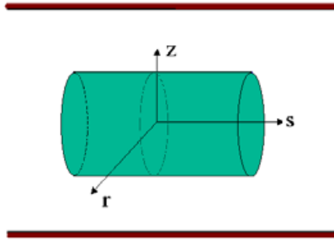
beam radius without space charge

$$a_0^2 = \frac{\varepsilon R}{\upsilon_0}$$

Increase in beam radius with beam current

$$a^2(I) = a_0^2 [u + \sqrt{1 + u^2}]$$

$$u = \frac{K R}{2 \varepsilon \upsilon_0}$$



Decrease in tune value with beam current

$$\upsilon(I) = \upsilon_0 [\sqrt{1 + u^2} - u]$$

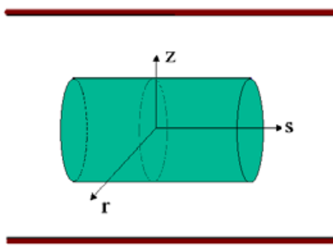
Transverse limit due to space charge is reached if

- Tune υ gets depressed to a dangerous resonance value
- Amplitude becomes too large

T SC Effects: Summary

• Transverse space charge effect increases the beam size and responsible for axial beam loss (serious at low energies).

$$I_m = \frac{I_0}{2}\beta^2\gamma^2\varepsilon_n \frac{\upsilon_0}{R}\cdot\frac{\Delta\phi}{2\pi}$$



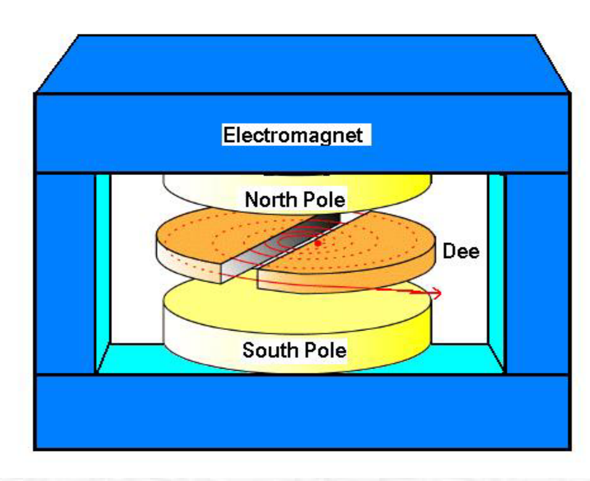
⇒Possible solutions:

a. inject beam at high energy (~100keV)

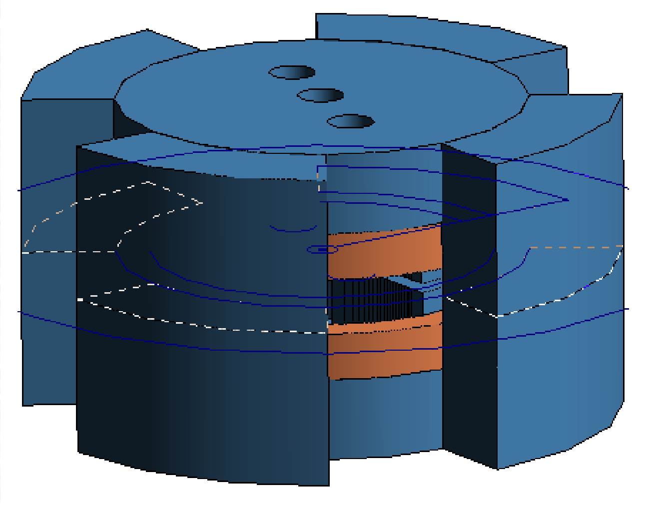
b. use large injection radius

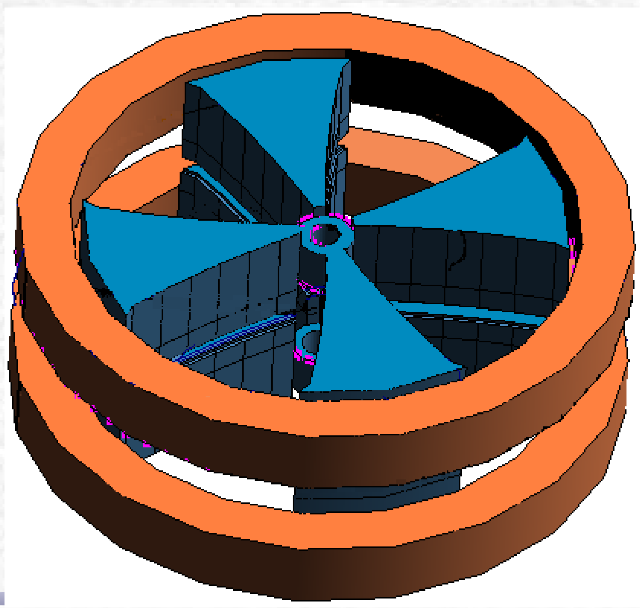
c. provide sufficient axial focusing

d. use high energy gain per turn



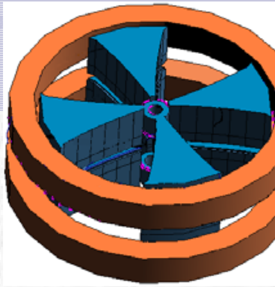
T SC Effects: Injector Cyclotron





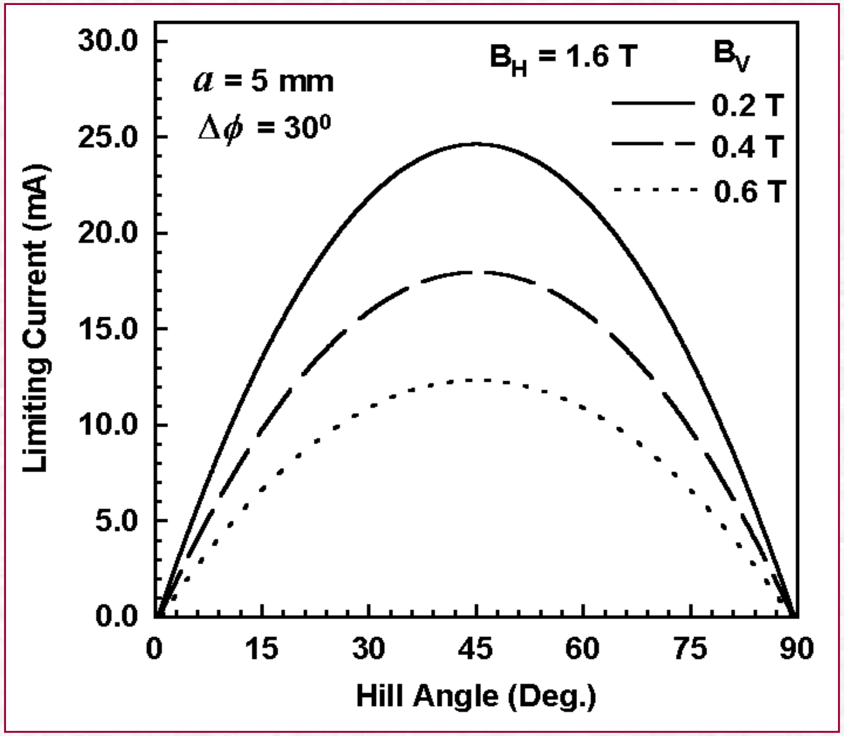
Injection Energy	100 keV
Final Energy	10 MeV
Hill Field B_H	1.5 T
Valley Field B_V	0.15 T
Pole gap Hill /Valley	4 cm / 66 cm
Hill angle max.	34.2^0
No. of resonators	2 Δ type, 45^0
RF Voltage inj. /extr.	125 / 150 kV
Injection radius	>6.6 cm
Phase width	< 30^0
Radial tune υ_r	1.1 - 1.2
Vertical tune υ_z	0.61 - 0.99
Beam current	5mA
Turn separation	6mm @5mA
I (limiting) TSC / LSC	15mA/13mA

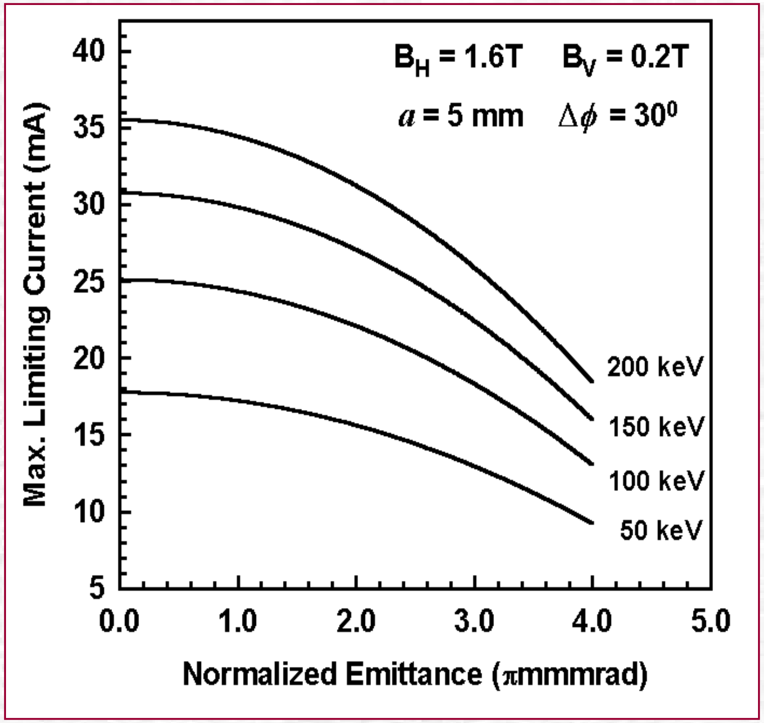
Estimation of Limiting current



$$\upsilon_{z\,max}^{2} = -\beta^{2}\gamma^{2} + \frac{N^{2}}{N^{2}-1}\frac{(B_H - B_V)^2}{4B_H B_V}$$

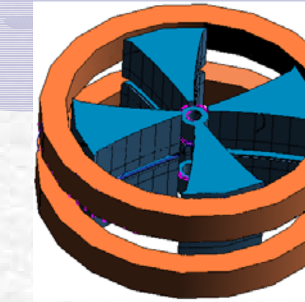
$$I_{max} = \frac{I_0\beta^3\gamma^3}{2}\frac{\Delta\phi}{2\pi}\times\left[\frac{q^2a^2(B_H+B_V)^2}{4m^2c^2\beta^2\gamma^2}\left(-\beta^2\gamma^2+\frac{N^2}{(N^2-1)}\frac{(B_H-B_V)^2}{(B_H+B_V)^2}\right)-\frac{\varepsilon_n^2}{a^2}\right]$$





V.S. Pandit, P.S. Babu; **NIM A523** (2004) 19-24 (for details)

Beam Envelope at Injection



- Hills & valleys are treated as bending magnets.
- For flaring & edge effect we used thin lenses at H-V boundary.
- Four gaps with 100 kV each..

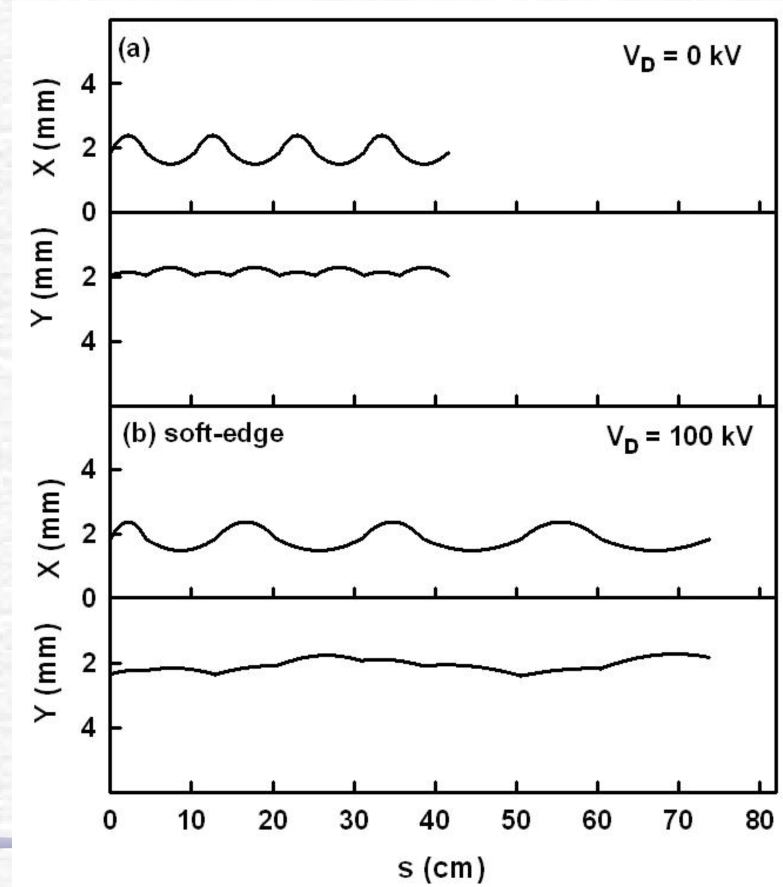
$\varepsilon_n(=\beta\gamma\cdot\varepsilon_x=\beta\gamma\cdot\varepsilon_y)=0.8\ \pi$ mmmrad

$$X'' + k_x^2 X - \frac{4I}{(X+Y)I_0\beta^3\gamma^3}\cdot\frac{2\pi}{\Delta\phi} - \frac{\varepsilon_x^2}{X^3} = 0$$

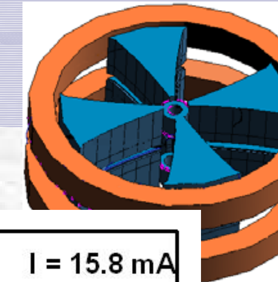
$$Y'' - \frac{4I}{(X+Y)I_0\beta^3\gamma^3}\cdot\frac{2\pi}{\Delta\phi} - \frac{\varepsilon_y^2}{Y^3} = 0$$

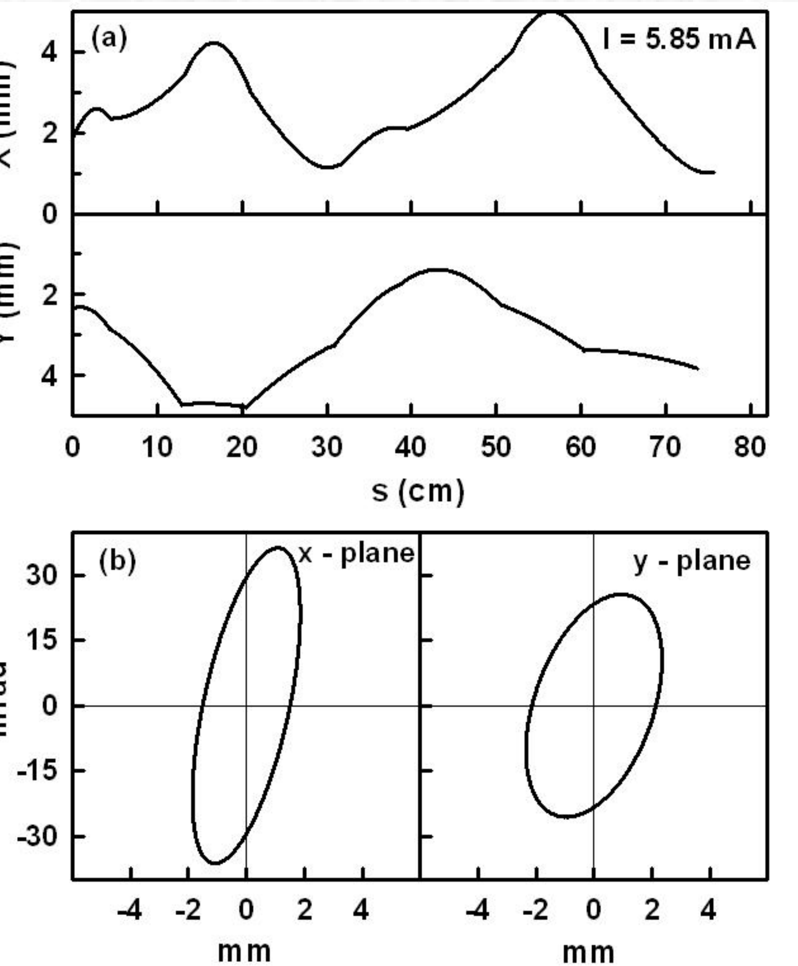
$$\beta_x = \frac{X^2}{\varepsilon_x},\quad \alpha_x = -\frac{XX'}{\varepsilon_x},\quad \gamma_x = \frac{1+\alpha_x^2}{\beta_x}$$

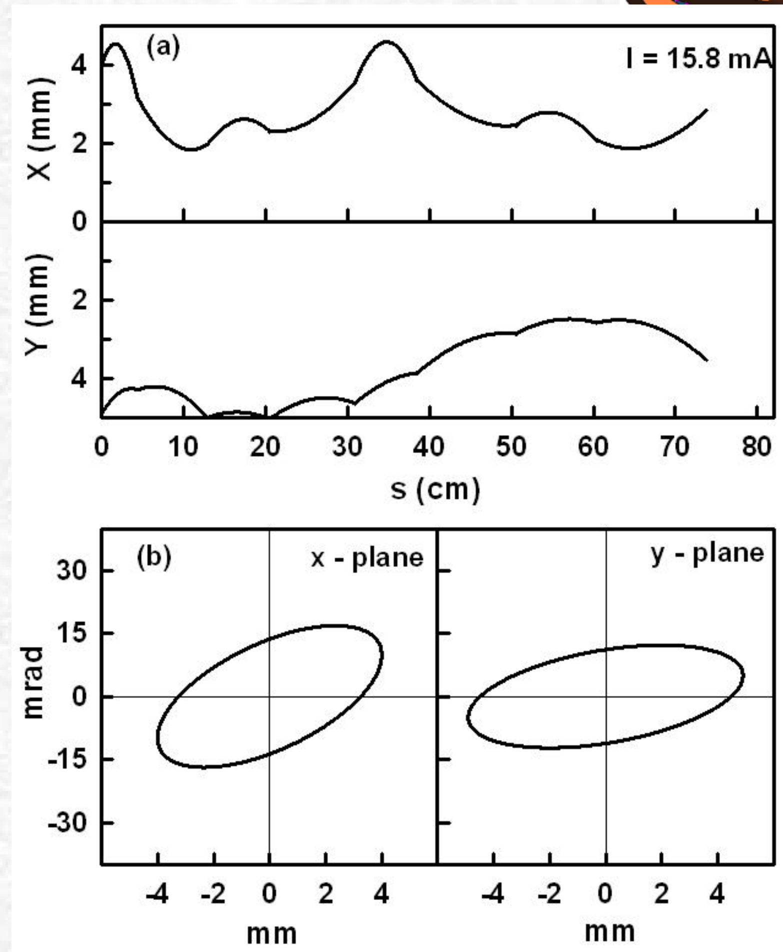
$$J_2 = R\cdot J_1\cdot R^{-1},\quad J = \begin{bmatrix}\alpha & \beta \\ -\alpha & -\gamma\end{bmatrix}$$



Beam Envelope at Injection

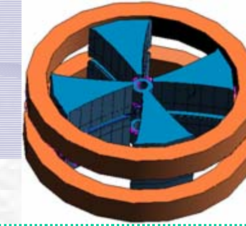






Possible to injected I ~10 mA within 10mm aperture)

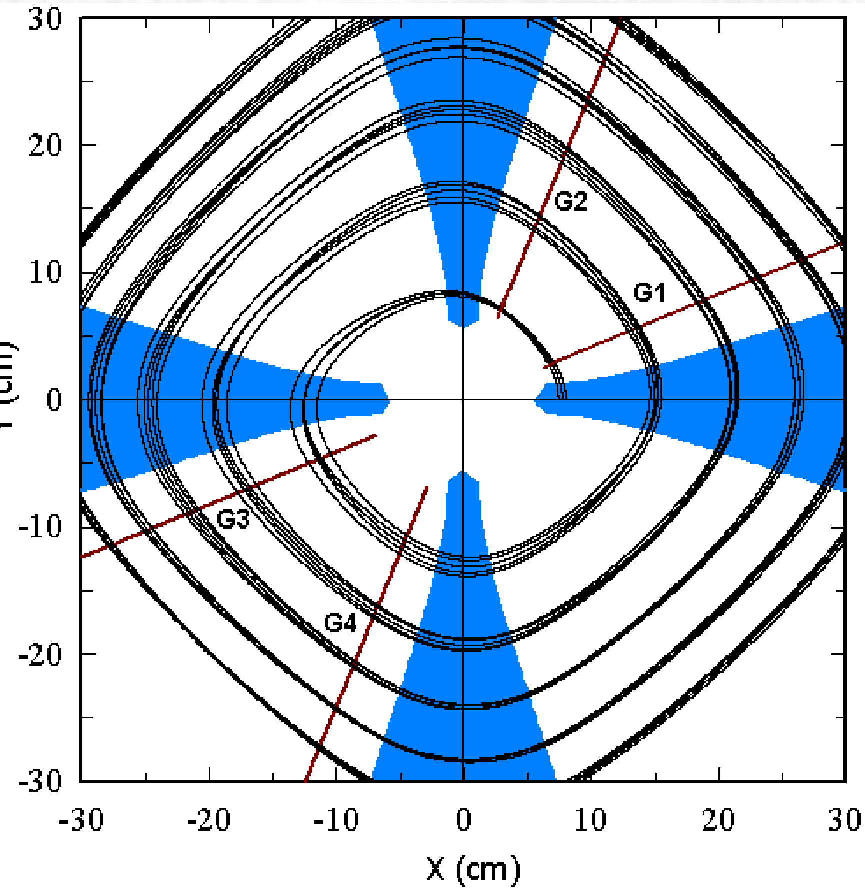
A. Goswami, P.S. Babu V.S. Pandit; **NIM A 562** (2006)34 (for details)

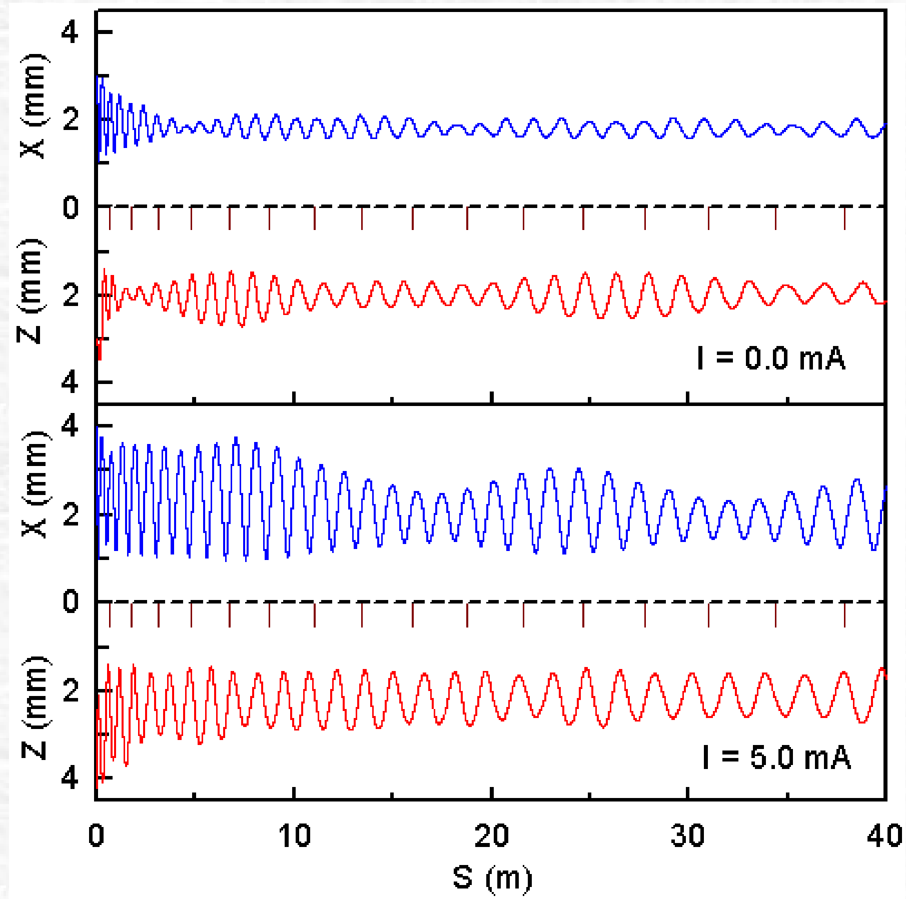


Beam envelop in the cyclotron

$$\frac{X''}{R^2}+\frac{\nu_r^2}{R^2}X-\frac{4I}{(X+Z)I_0\beta^3\gamma^3}\cdot\frac{2\pi}{\Delta\phi}-\frac{\varepsilon_x^2}{X^3}=0$$

$$\frac{Z''}{R^2}+\frac{\nu_z^2}{R^2}Z-\frac{4I}{(X+Z)I_0\beta^3\gamma^3}\cdot\frac{2\pi}{\Delta\phi}-\frac{\varepsilon_z^2}{Z^3}=0$$

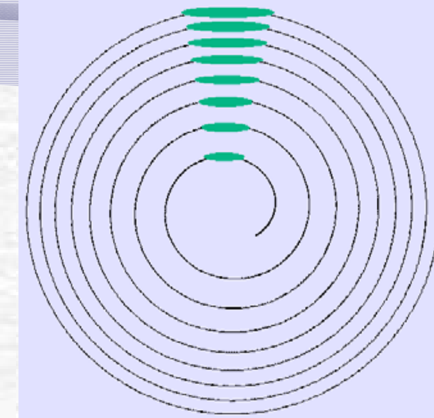




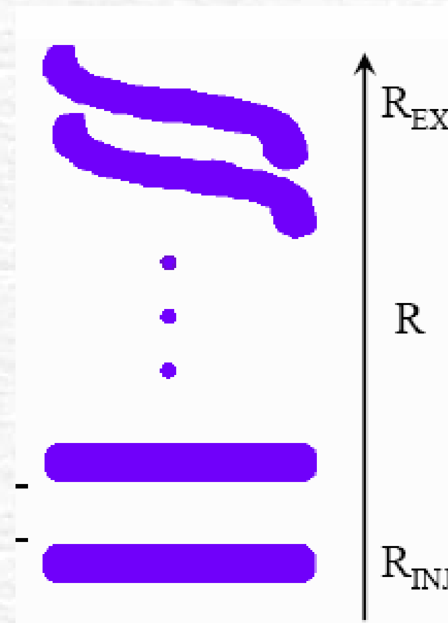
Longitudinal Space Charge Effects

Longitudinal space charge introduces extra energy spread in the beam and must be added to the beam energy.

$$p = qBr$$



- Cyclotrons do not have longitudinal focusing
- Leading particles gain energy and drift outwards.
- Late particles loose energy and drift inwards.
- These motions results in a rotation of bunch.
    - LSC increases effective radial size.
    - LSC destroys turn separation.
    - LSC causes loss in extraction deflector.

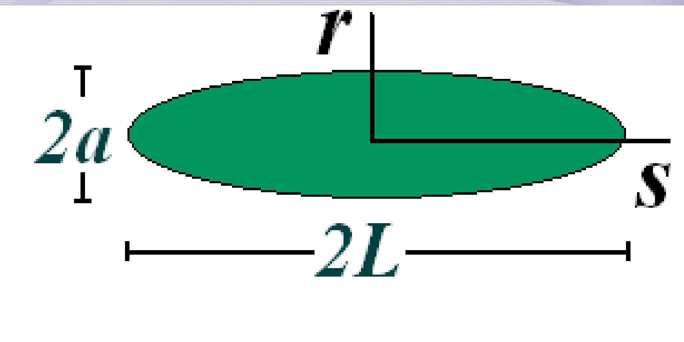


- Linear part can be estimated and controlled.

The nonlinear part results in a deterioration of beam quality, creation of long tails and increasing beam loss.

L SC effects: Uniform beam model

- Beam bunch is a prolate spheroid (cigar model).
- Particle distribution is uniform.
- Effect of neighbouring orbit is neglected



Free space potential in the bunch frame satisfies

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_0}$$

$$Q = \frac{4}{3}\pi a^2 L\cdot\rho \qquad I_p = \frac{Q}{t} = \frac{Qv}{2L}$$

Solution is

$$\phi(r,s) = -\frac{\rho}{2\varepsilon_0}\left(\frac{1-M}{2}r^2 + Ms^2\right)$$

$$M = \frac{1-\xi^2}{\xi^2}\left(\frac{1}{2\xi}\cdot\ln\frac{1+\xi}{1-\xi} - 1\right)$$

$$\xi = \sqrt{1-(a/L)^2}$$

$$E_r = \frac{\rho}{2\varepsilon_0}(1-M)r = \frac{\rho}{2\varepsilon_0}\left(1-\frac{a}{3L}\right)r$$

$$E_s = \frac{\rho}{\varepsilon_0}Mz = \frac{\rho a}{3\varepsilon_0 L}z$$

When

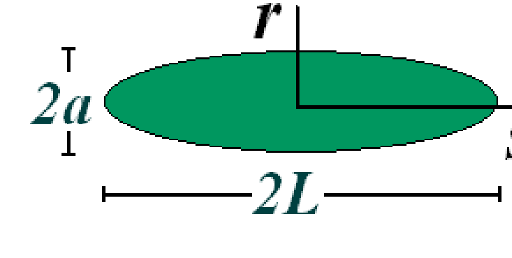
$$a = L, \quad M = 1/3$$

$$.8 \le \frac{L}{a} < 5, \quad M \approx \frac{a}{3L}$$

L SC effects: energy spread

Electric field seen by the front particle

$$E_s = \frac{\rho a}{3\varepsilon_0 L} z = \frac{\rho a}{3\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 a L} = \frac{I_p}{2\pi\varepsilon_0 a v}$$



$$Q = \frac{4}{3}\pi a^2 L \cdot \rho$$

$$I_p = \frac{Q}{t} = \frac{Q}{2\ldots}$$

$$I \times 2\pi = Ip \times \Delta\phi$$

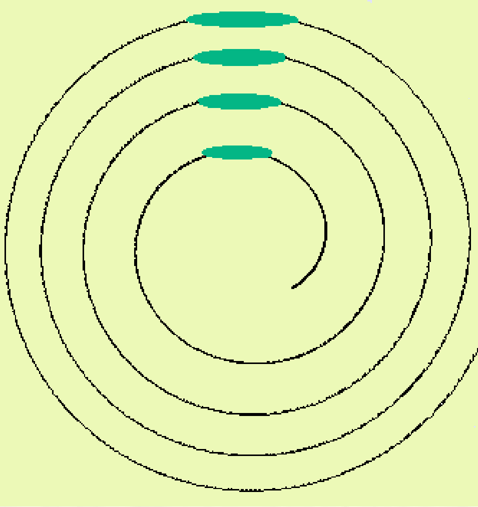
$$v = \omega R = 2\pi f R$$

Leading and lagging particles both faces such field

Leading particles gain energy
Lagging particles lose energy

Energy spread over one turn in lab

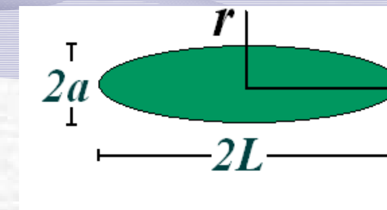
$$\frac{dU_{sp}}{dn} = \frac{2}{\gamma^2} \cdot 2\pi R \cdot qE_s = \frac{2qI}{\varepsilon_0 \Delta\phi f} \cdot \frac{1}{a\gamma^2}$$



Total energy spread over n revolutions

$$\Delta U_{sp} = \frac{2qnI}{\varepsilon_0 \Delta\phi f} < \frac{1}{a\gamma^2} >$$

L SC effects: Limiting current



When total energy spread becomes equal to energy gain per turn due to rf field, turns overlap.

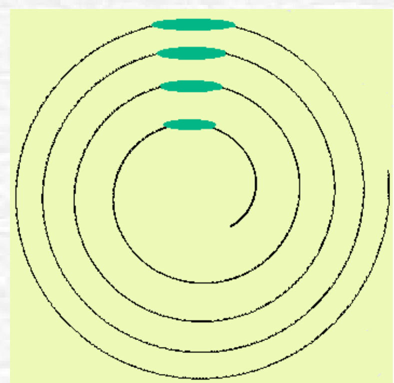
$$p = qBr$$

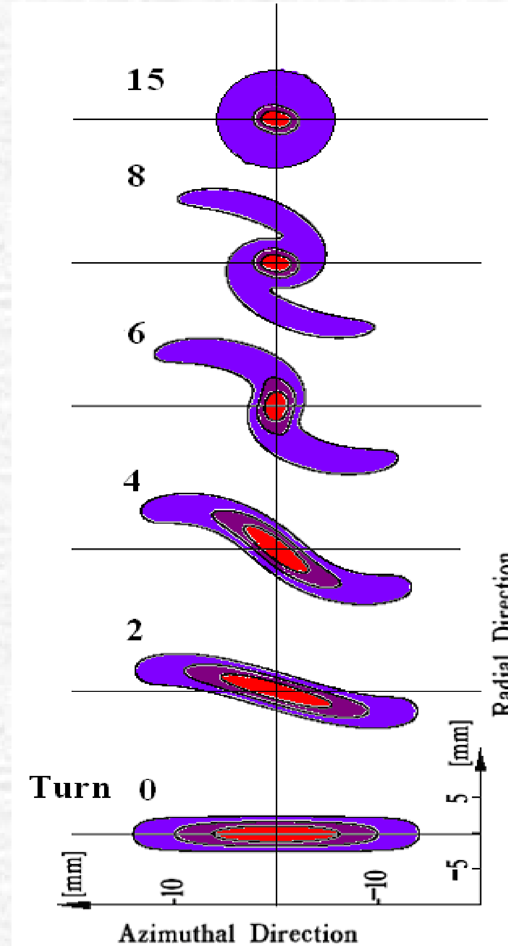
Condition for turn overlapping

$$\Delta U_{sp} = \frac{2qnI}{\varepsilon_0 \Delta\phi f} < \frac{1}{a\gamma^2} >= \frac{T}{n}$$

Limiting current is

$$I_{Lim} = \frac{T\varepsilon_0 \Delta\phi f}{2qn^2} \left(< \frac{1}{a\gamma^2} >\right)^{-1}$$





PICS of 15° phase width

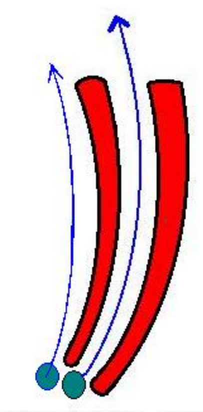
Possible solutions

- Use high energy gain per turn to reduce no of turns
- Use strongly bunched beam

Extraction and LSC

In a cyclotron main aim is to achieve high extraction efficiency efficiency

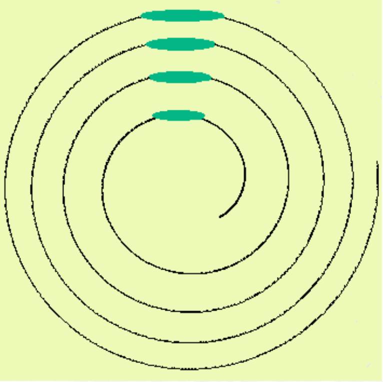
Last turn must be separated from the previous turns for a septum.



To Improve turn separation

$$d = \frac{dR}{dn} = R\frac{\Delta T}{T}\frac{\gamma}{\gamma+1}\frac{1}{\nu_r^2}$$

- Use more numbers RF resonators,
- Use high voltages on the resonators
- Place the extraction system where radial tune is low.



The effects that reduce the turn separation are

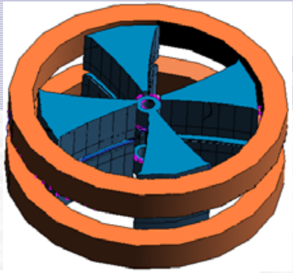
RF phase width of the beam $\Delta R_\phi = d \cdot n \cdot (1 - \cos\phi)$

Injected beam energy spread $\Delta R_e = (w_i + dR_i)R_i / R_{ext}$

Stability of RF voltage and magnet current $\Delta R_{rf} = nR_i(dV / V_D)$

Longitudinal space charge effect $\Delta R_{LSC} = R_{ext}\Delta U_{sp} / (2.T_{ext})$

Extraction and LSC

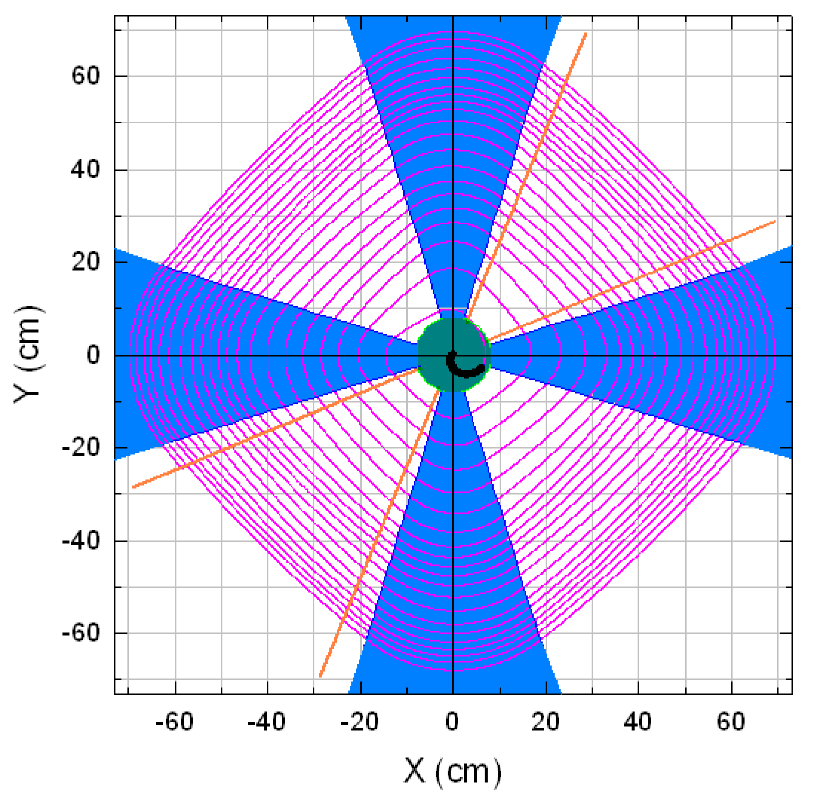


R_{inj} =6.5 cm R_{ext} = 65 cm

V_{dee} =150 kV at ext

Effective turn separation @ 10 MeV

ΔR (acceleration)	= + 24.5 mm
ΔR (ΔE initial)	= - 0.4 mm
ΔR (Phase ϕ =30^0)	= - 13.0 mm
ΔR (LSC ~.18MeV)	= - 5.0 mm

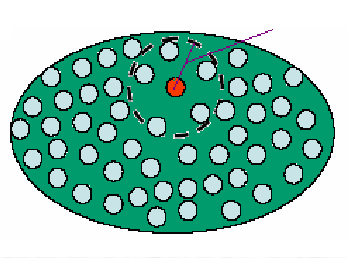


Effective turn separation

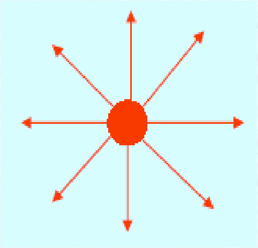
ΔR = 6.0 mm @ 5mA

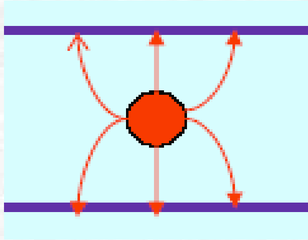
ΔR= 0.77mm @ 8mA

Summary

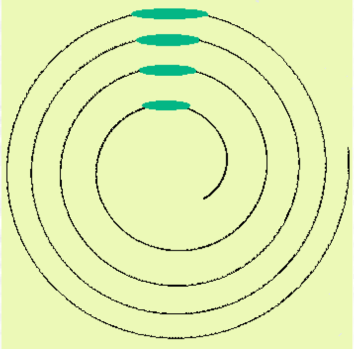


- In this talk we have studied only linear beam dynamics. Results presented here are only rough estimates.
  - Electric field depends on the charge distribution especially on the periphery of the bunch.



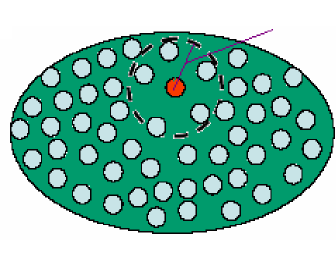


  - Shielding effect on the walls above and below the bunch changes the electric field.
  - Neighboring orbits contribute to the longitudinal electric field.

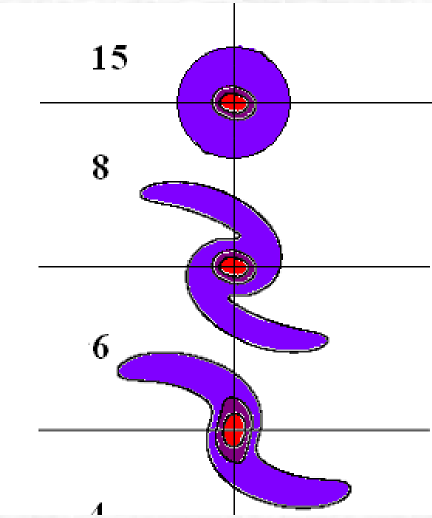


- However, these give useful scaling laws.
- In cyclotrons longitudinal space charge effect dominates because
  - there is no longitudinal focusing.
  - there is a strong coupling between longitudinal and radial motions.

Summary



- The relative motion of the particles in the bunch must be separated from the motion of the bunch as a whole.
- The relative motion then changes the particle distribution that defines the field.



- The space charge force and particle distribution have to be treated in a self consistent manner.
- Resulting forces are highly nonlinear.
- Study of tails and halos, which are major sources of beam loss, needs simulations with large no of particles.
- Tails of the profiles are determined by non-linearity and for this one needs actual charge distribution in the bunch which hard to predict.
- An accurate prediction of the behavior of beam losses is difficult.

Thanks